DAWSON COLLEGE

Mathematics Department

FINAL EXAMINATION

Calculus I- 201-NYA-05 (Open/Commerce)

Fall 2021

Instructor: Noushin Sabetghadam

Student Name: ______________________________

Student ID. #: ______________________________

Instructions:

- Print your name and ID in the provided space.
- Solve the problems in the space provided for each question and show all your work clearly and indicate your nal answer(s).
- Only calculators Sharp EL 531.X/ XG/XT are permitted.
- This examination booklet must be returned intact.

This examination consists of 11 questions. Please ensure that you have a complete examination booklet before starting.

(1) Evaluate the following limits if possible and **write all the details.**(No marks is given if you use L'hopital Rules)

(a) (5 marks) $\lim_{x \to 1} \dfrac{x - \frac{1}{x}}{x^2 - 8x + 7}$

(b) (4 marks) $\lim_{x \to 0} \dfrac{\sin(2x) \tan(3x)}{4x^2}$

(c) (6 marks) $\lim_{x \to -3} \dfrac{\sqrt{x^2 + 7} - 4}{x^2 + 3x}$

(2) **(5 marks)** Use only the limit de nition of the derivative to evaluate $f'(x)$ where

$$f(x) = 3x^2 \quad 2x \quad 5$$

(3) **(4 marks)** **(a)** Evaluate For which value(s) of a the function f is continuous everywhere?

$$f(x) = \begin{cases} ax^2 + x & \text{if } x \quad 1 \\ \dfrac{3a}{x+1} & \text{if } x > \end{cases}$$

(4) (20 marks) Find the derivative. (Do NOT simplify)

(a) $y = \frac{2x^3 + 5x}{}$

(d) $g(x) = 3^{(2x+1)} \quad \arctan(x^2)$

(5) If $sin(y) + 3x^2y^3 = 5x \quad y$, then:

(a) (5 marks) nd $\dfrac{dy}{dx}$

(b) (2 marks) Find the tangent line to the graph of the given function at the point $(0;0)$.

(7) (

(8) (6 marks) A rectangular closed storage container is to have a base whose length is two times its width and a volume of 1440cm^3. If the material for the base and top costs \$0:25 per cm^2 and the material for the sides costs \$0:10 per cm^2, nd the cost for the cheapest such container.

(9) **(5 marks)** Find the point(s) on the graph of the function $y = \dfrac{x^3}{(x^2 \quad 1)^2}$ where the tangent line is horizontal.

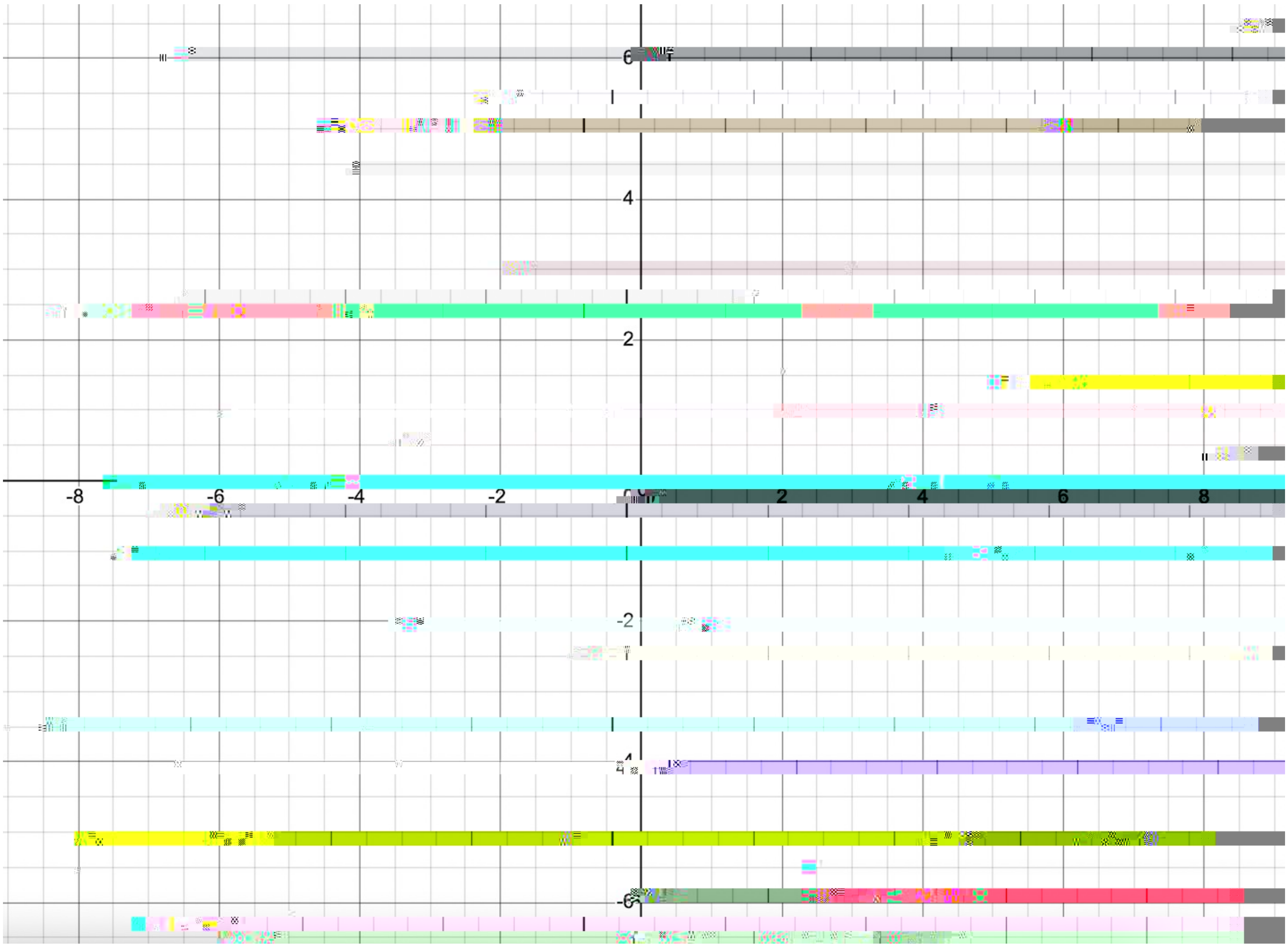
(10) **(5 marks)** Let $f(x) = \dfrac{\ln x}{x}$, Find $f''(x)$ and then evaluate $f''(e)$.

(11) Given the function $f(x) = \dfrac{x^2 \quad 2}{4}$

(d) **(3 marks)** Find the **intervals** where the function f is increasing or decreasing and also nd its **relative maximum point(s) and relative minimum point(s)** if there is any.

(e) **(3 marks)** Find the **intervals** where the function f is concave upward or downward and also nd its **in ection point(s)** if there is any.

(f) **(2 marks)** Use all the data collected about f to draw the its graph in the coordinate system below.



Final Answers

(1) (a) $\frac{1}{3}$

(b) $\frac{3}{2}$

(c) $\frac{1}{4}$

(2) $f'(x) = 6x \quad 2$

(3) (a) $a = 2$

(b) $\frac{7}{3}$

(4) **(a)** $y' = 3 \left(\frac{2x^3 + 5x}{\sqrt{3x^5 + 2x}}\right)^2 \frac{(6x^2 + 5)(\sqrt{3x^5 + 2x} \quad \frac{15x^4 + 2}{2\sqrt{3x^5 + 2x}}(2x^3 + 5x)}{(3x^5 + 2x)}$

(b) $f'(x) = \quad 2\cos(x^2 + 3)2x\sin(x^2 + 3) + 3\sec^2(\log_2 x)\frac{1}{x\ln 2}$

(c) $y' = (1 + \ln x)^{\sec x}[\sec x \tan x \ln(1 + \ln x) + \sec x \frac{\frac{1}{x}}{1 + \ln x}]$

(d) $g'(x) = 2\ln 3 \quad 3^{(2x+1)} \quad \arctan(x^2) + \frac{2x}{1 + x^2} \quad 3^{(2x+1)}$

(5) (a) $y' = \frac{5 \quad 6xy^3}{\cos y + 9x^2y^2 + 1}$

(b) $y = \frac{5}{2}x$

(6) (a) $P(x) = \quad 0.004x^3 + 0.9x^2 + 1200x \quad 16000$

(b) 1257.06$

(c) 1257.41$

(7) $\frac{dx}{dt} \quad 48.97$ mph

(8) 108$

(9) at (0.0)

(10) $f''(x) = \dfrac{2\ln x - 3}{x^3}$; $f''(e) = -\dfrac{1}{e^3}$

(11) (a) $D(f) = \mathbb{R} \setminus \{-2; 2\}$

(b) $(-\sqrt{2}; 0)$; $(\sqrt{2}; 0)$; $(0; -1/2)$

(c) $x = -2$; $x = 2$; $y = -1$

(d) f is increasing on $(0; 2) \cup (2; \infty)$ and decreasing on $(-\infty; -2) \cup (-2; 0)$. There is a relative min at $(0; -1/2)$.

(e) f is concave upward on $(-2; 2)$ and downward on $(-\infty; -2) \cup (2; \infty)$. There is no inflection point.

(f)