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Note:

- Write your ans
- Only Sharp EL
- Use the reverse
- The exam has

Reserved for Marking

Q1	Q2	Q3	Q4
/12	/6	/4	/3

1. []
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$$(c) \lim_{x \rightarrow 4} \frac{x^2 - x - 12}{|x - 4|}$$

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x

.n.e.

↓

$$- \frac{x^2 - x - 12}{|x - 4|} =$$

$$\frac{x^2 - x - 12}{|x - 4|} = l$$

$$(d) \lim_{x \rightarrow \infty} \arctan\left(\frac{1}{x}\right)$$

$$= \arctan \lim_{x \rightarrow \infty}$$

2. [6 marks] Given the function f :

$$f(x) = \begin{cases} \frac{2x^2}{x-1} \\ \dots \end{cases}$$

Find the x -values where f is discontinuous in each case. Refer to the definition of continuity.

• $x = 1$ JUMP DISCONTINUITY

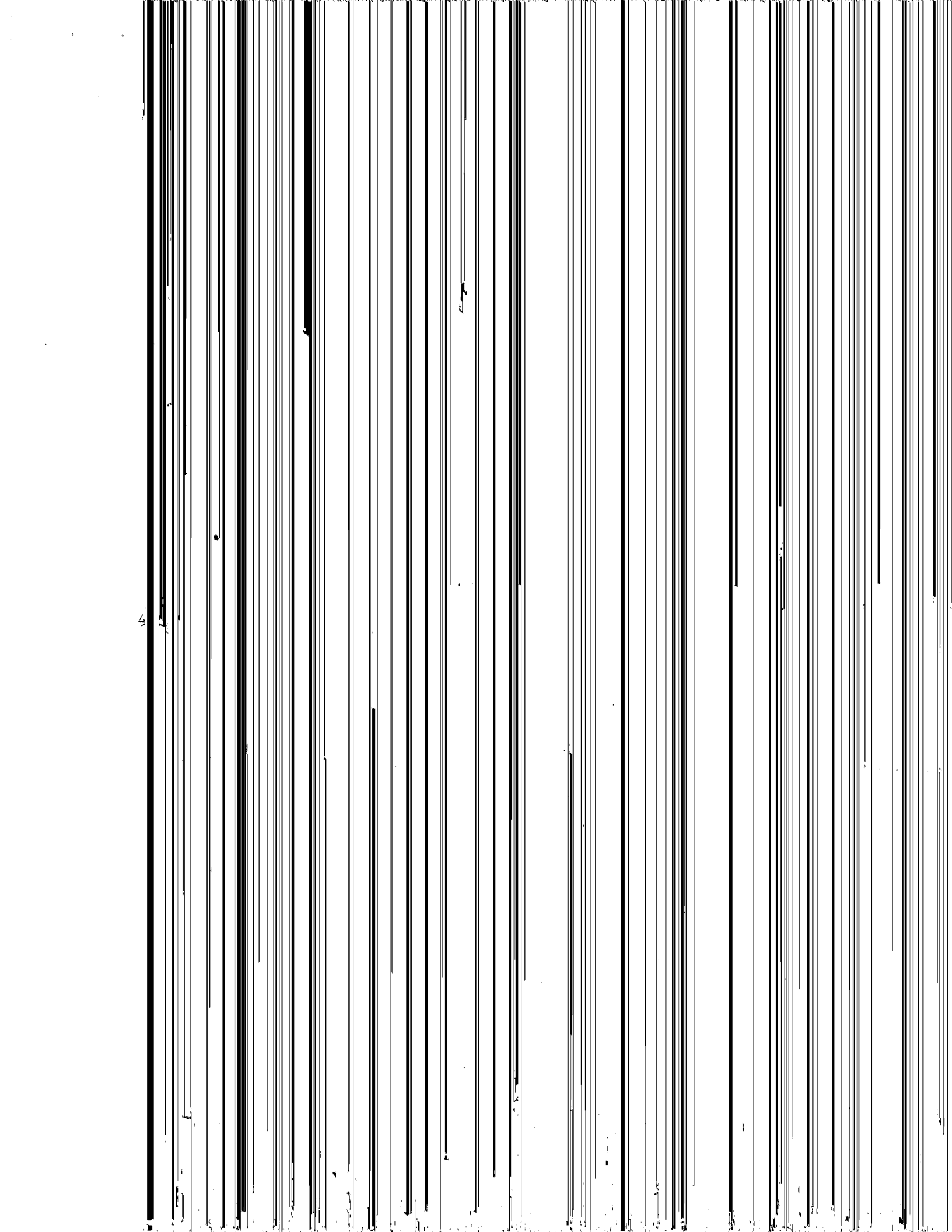
because $\lim_{x \rightarrow 1} f(x)$ d.n.e.

• $x = -1$ HOLE (REMOVABLE)

because $f(-1)$ d.n.e., $\lim_{x \rightarrow -1} f(x)$ exists

• $x = 2$ INFINITE DISCONTINUITY

because $f(2)$ d.n.e. $\lim_{x \rightarrow 2} f(x)$ does not exist



5. [12 marks] Find the derivatives for the

$$(a) y = \frac{\sqrt{3x-4}}{(5x+2)^5}$$

$$y' = \frac{\frac{1}{2\sqrt{3x-4}} \cdot 3 \cdot (5x+2)^5}{(5x+2)^{10}}$$

$$(b) y = \arctan(\pi 5^{3x}) \sec(4x) + \cos^2\left(\frac{1}{\csc x}\right)$$

$$y' = \frac{1}{1+(\pi 5^{3x})^2} \cdot \pi \cdot 5^{3x} \ln 5$$

$$\cdot \sec(4x) \tan(4x) \cdot 4 + 2 \cos x$$

$$\cdot (-\csc x) (\cot x)$$

(c) y

Y

6. [4 marks]

$$y = x^{\frac{1}{2}}$$

$$\ln y = \frac{1}{2} \ln x$$

$$y' = \frac{1}{2x}$$

$$y'' = -\frac{1}{2x^2}$$

7. [3 marks]
(in degrees)

(a) Work

T

(b) How
take

T'

T'

(c) Find

$\lim_{t \rightarrow \infty}$

After

This

8. [5 marks] Given $f(x) = 2 \arctan x$ and $g(x) = \ln(1 + x^2) + \frac{\pi}{2} - \ln 2$, find whether they have any common tangent line at one and the same point, and if yes, find the equation of the tangent(s).

$$f'(x) = g'(x)$$

$$\frac{2}{1+x^2} = \frac{2x}{1+x^2}$$

$$2x = 2$$

$$x = 1$$

$$f(1) = g(1) = \frac{\pi}{2}$$

Point of common tangent $(1, \frac{\pi}{2})$

Slope of the tangent: $f'(1) = g'(1) = 1$

Equation of the tangent

$$y = ax + b$$

$$\frac{\pi}{2} = 1 + b$$

$$b = \frac{\pi}{2} - 1$$

$$\boxed{y = x + \frac{\pi}{2} - 1}$$

9. [5 marks] I

at the poin

$$\frac{y + xy}{xy}$$

Replace

$$1 + y'$$

$$y' | (1, 1)$$

$$y = a$$

$$1 = 1 \cdot 1$$

$$b = 0$$

$$\boxed{y = x}$$

012 21

11. [3 marks]
give a cou

(a) Ration

False

is m

(b) Polyn

True

and c

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(c) Any ir

False

$x=0$

12. [8 marks] Find the

$$(a) \lim_{x \rightarrow 0} \left(\frac{1}{\ln(x+1)} \right)$$

$$= \lim_{x \rightarrow 0} \frac{x-0}{x}$$

$$\begin{array}{l} \frac{0}{0} \\ = \\ \text{L'H} \end{array} \lim_{x \rightarrow 0} \frac{\frac{1}{x+1}}{\frac{x+1}{x}}$$

$$(b) \lim_{x \rightarrow 0^+} e^{\frac{1}{x}} \ln x$$

$$(c) \lim_{x \rightarrow 0^+} (2^x - 1)^x \quad 0^0$$

$$= e^{\lim_{x \rightarrow 0^+} \ln (2^x - 1)^x} = e^{\lim_{x \rightarrow 0^+} x \ln (2^x - 1)} = e^L$$

$$L = \lim_{x \rightarrow 0^+} \frac{\ln(2^x - 1)}{\frac{1}{x}} \stackrel{\frac{0}{0}}{=} \lim_{x \rightarrow 0^+} \frac{2^x \ln 2}{-\frac{1}{x^2}}$$

$$= -\ln 2 \lim_{x \rightarrow 0^+} \frac{2^x x^2}{2^x - 1} \stackrel{\frac{0}{0}}{=} -\ln 2 \lim_{x \rightarrow 0^+} \frac{\cancel{2^x} \ln 2 x^2 + \cancel{2^x} \cdot 2x}{\cancel{2^x} \ln 2} =$$

$$= -\ln 2 \cdot 0 = 0$$

$$\lim_{x \rightarrow 0^+} (2^x - 1)^x = e^0 = 1$$

13. [15 marks] Consider the function:

$$f(x) = \frac{(1+x)^4}{(1-x)^4} \text{ with } f'(x) = \frac{8(1+x)^3}{(1-x)^5} \text{ and } f''(x) = \frac{16(1+x)^2(x+4)}{(1-x)^6}.$$

(a) Find its x - and y -intercepts.

$$x\text{-int. } y=0 \Rightarrow x=-1 \quad (-1, 0)$$

$$y\text{-int. } x=0 \Rightarrow y=1 \quad (0, 1)$$

(b) Find its asymptotes, if any. Justify your answer using limits.

$$\text{Vertical } x=1 \quad \lim_{x \rightarrow 1} \frac{(1+x)^4}{(1-x)^4} = +\infty$$

$$\text{Horizontal } y=1 \quad \lim_{x \rightarrow \pm\infty} \frac{(1+x)^4}{(1-x)^4} = 1$$

(c)

$\frac{1}{2}$
 $\frac{1}{4}$
 $\frac{1}{8}$

f
 f

(d)

f''

x	1
f''	
f	

f con
 f con
Point

$$y = 1$$

14.

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15. [6 marks] Find the antiderivatives:

$$(a) \int 3x^2 \left(\frac{4}{x^3} + \frac{5}{\sqrt{x}} + 8x^7 \right) dx$$

$$= \int (12x^{-1} + 15x^{3/2} + 24x^9) dx$$

$$= 12 \ln|x| + 6x^{5/2} + \frac{12}{5} x^{10} + C$$

$$(b) \int (\sin x \cos x) e^{\sin^2 x} dx = \frac{1}{2} e^{\sin^2 x} + C$$

16. [4 marks] Given

$$f'(x) = \int (x)$$

$$f'(0) = 0$$

$$f(x) = \int (-x)$$

$$f(0) = 0$$

$$\text{So } f(x) = -\frac{1}{2}x^2$$