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Calcu
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Mahendra Chaubey, Oxana
Alexander Hariton, Yar
Benjamin Seamone,

Name: _____ S

Student ID:

- There are a total of 100 marks on th
- There are a total of 12 pages on this
- Show all your work unless indicated ot
marks.
- You may use the back of the pages to
- Do not remove any pages from the exa

1. Find $\int_{-4}^2 \left(-3 - \frac{3}{2}x\right)$

[4 pt]

(a) taking the limit

[2 pt]

(b) interpreting the

Summation formulas

$$\sum_{i=1}^n c = cn$$

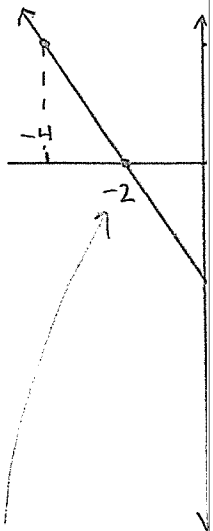
(a) $\Delta x = \frac{b}{n}$,

$$\int_{-4}^2 \left(-3 - \frac{3}{2}x\right)$$

(b) $f(x) = -3 -$

$$f(-4) = 3$$

$$f(2) = -6$$



$$-3 - \frac{3}{2}x = 0$$

$$\Rightarrow x = -2$$

2. Eval

[5 pt]

(a)

\Rightarrow

\uparrow

[5 pt]

(b)

[5 pt]

$$(c) \int \frac{1}{v(v+3)^2} dv = \int \left(\frac{A}{v} + \frac{B}{v+3} + \frac{C}{(v+3)^2} \right) dv$$

$$1 = A(v+3)^2 + B(v+3) + C$$

$$v=0 \Rightarrow 1 = 9A$$

$$v=-3 \Rightarrow 1 = -3C$$

$$v=-2 \Rightarrow 1 = A -$$

$$\int \frac{1}{v(v+3)^2} dv = \int \left(\frac{1}{9v} + \frac{-1}{3(v+3)} + \frac{1}{(v+3)^2} \right) dv$$

$$= \frac{1}{9} \ln |v| - \frac{1}{3} \ln |v+3| - \frac{1}{v+3} + C$$

$$= \ln \left| \frac{v}{(v+3)^3} \right| + C$$

[5 pt]

$$(d) \int e^{\sqrt{3x+1}} dx$$

$$= \int e^t \cdot \frac{2}{3} t dt$$

$$= \frac{2}{3} \int t e^t dt$$

By parts \longrightarrow

$$= \frac{2}{3} \left(t e^t - \int e^t dt \right)$$

$$= \frac{2}{3} t e^t - \frac{2}{3} e^t + C$$

$$= \frac{2}{3} \sqrt{3x+1} e^{\sqrt{3x+1}} - \frac{2}{3} e^{\sqrt{3x+1}} + C$$

[5 pt]

3. Fir

70

[5 pt]

4. Fir

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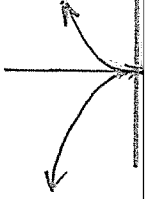
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[5 pt]

5. Deter



[5 pt]

6. Find th

5

7

[8 pt.]

7. Let \mathcal{R} be

(a) Stat
 \mathcal{R} al

(b) Stat
 \mathcal{R} al

(a)

$y=1$

(a)

(b)

$2-y$

8. In all

[2 pt] (a) I
(

[2 pt] (b) I

[1 pt] (c) I

(

(

(

[3 pt] 9. Suppo

$$\int_0^0$$

$$\lim_{x \rightarrow 0^+}$$

] 10. Evaluate

a

$\lim_{n \rightarrow a}$

11. Suppose
integral

Determine

$f(x)$

\therefore

$=$

12. (a) Show that $0 < \frac{1}{2}$
 (b) Use part (a) (whether it converges or diverges. If it

$$(a) \frac{(n!)^2}{(2n)!} >$$

$$\frac{(n!)^2}{(2n)!} =$$

(b)

$$\therefore a_n$$

13. Determine if each of the

(a) $\pi - \frac{2\pi}{e} + \frac{4\pi}{e^2} - \frac{8\pi}{e^3} + \dots$

Geometric

Since $|r| < 1$

$$S = \frac{a}{1-r}$$

(b) $\sum_{n=1}^{\infty} a_n$ where the

$$\sum_{n=1}^{\infty} a_n = \lim_{n \rightarrow \infty} a_n$$

\therefore the

14. Determine if each of the following series converges or diverges. Clearly state which test you use and your conclusion.

[4 pt]

$$(a) \sum_{n=2}^{\infty} \frac{1}{n \sqrt[3]{\ln n}}$$

Integral Test

$$\bullet f(x) \geq 0 ?$$

$$\bullet f(x) \text{ cts?}$$

$$\bullet f(x) \text{ dec?}$$

$$\int_2^{\infty} \frac{1}{x \sqrt[3]{\ln x}} dx =$$

=

=

=

$$\text{Since } \int_2^{\infty} \frac{1}{x \sqrt[3]{\ln x}}$$

[4 pt]

$$(b) \sum_{n=6}^{\infty} \left(1 - \frac{5}{n}\right)^{n^2}$$

Root test (no)

$$\lim_{n \rightarrow \infty} \sqrt[n]{\left(1 - \frac{5}{n}\right)^{n^2}}$$

$$\Rightarrow \ln L = \lim_{n \rightarrow \infty}$$

$$= \lim_{n \rightarrow \infty}$$

$$= \lim_{n \rightarrow \infty}$$

$$= \lim_{n \rightarrow \infty}$$

$$= \lim_{n \rightarrow \infty}$$

$$= 1$$

[4 pt]

$$(c) \sum_{n=2}^{\infty} (-1)^{n+1} \sqrt{\frac{1}{n^2-2}}$$

Alternating series

$$a_n = (-1)^{n+1} \frac{1}{\sqrt{n}}$$

$$\textcircled{1} b_n \rightarrow 0?$$

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^2-2}}$$

$$\textcircled{2} b_n \text{ decreasing}$$

$$\therefore \sum a_n \text{ converges}$$

Now consider \sum

$$\frac{1}{\sqrt{n^2-2}} >$$

Since $\sum \frac{1}{n}$ diverges

Direct Comparison

Since $\sum a_n$ converges $\sum a_n$ is conditionally convergent

[4 pt] 15. (a) Find the interval of convergence.

[4 pt] (b) Show that the Taylor series converges to $f(x)$ on the interval of convergence.

[1 pt] (c) Use (a) and (b) to evaluate the series at $x = \frac{5}{2}$ and $x = \frac{1}{2}$.

(a) Ratio Test:

$$\lim_{n \rightarrow \infty} \left| \frac{(-1)^{n+1} (1)^{n+1}}{(-1)^n (1)^n} \right|$$

The series converges for $|x - 2| < 1$.

At $x = 2 \Rightarrow$ converges

At $x = 4 \Rightarrow$ diverges

Each of the endpoints is not included in the interval of convergence.

(b) $f^{(0)}(x) = (x-2)^{-2}$

$f^{(1)}(x) = -2(x-2)^{-3}$

$f^{(2)}(x) = 6(x-2)^{-4}$

$f^{(3)}(x) = -24(x-2)^{-5}$

$f^{(n)}(x) = (-1)^n \frac{2^n n!}{(x-2)^{n+2}}$

$f^{(n)}(2) = (-1)^n \frac{2^n n!}{0^{n+2}}$

$= (-1)^n \frac{2^n n!}{0}$

(c) $f\left(\frac{5}{2}\right) = \sum_{n=0}^{\infty} \frac{(-1)^n 2^n n!}{(5/2 - 2)^{n+2}}$

$\frac{1}{(5/2 - 2)^2} = \sum_{n=0}^{\infty} \frac{(-1)^n 2^n n!}{(1/2)^{n+2}}$

$\frac{1}{(1/2)^2} = \sum_{n=0}^{\infty} (-1)^n 2^{n+2} n!$