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Calcu
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Student ID:

- There are a total of **100 marks** on th
- There are a total of **12 pages** on this
- Show all your work unless indicated ot marks.
- You may use the back of the pages to
- Do not remove any pages from the exa

1. Find $\int_{-4}^{2}\left(-3-\frac{3}{2}x\right.$

[4 pt] (a) taking the limit

[2 pt] (b) interpreting the

Summation formulas

$$\sum_{i=1}^{n} c = cn$$

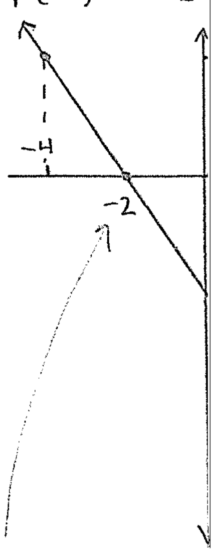
(a) $\Delta x = \frac{6}{n}$,

$\int_{-4}^{2}\left(-3-\right.$

(b) $f(x) = -3 -$

$f(-4) = 3$

$f(2) = -6$



$-3-\frac{3}{2}x = 0$

$\Rightarrow x = -2$

2. Eva

[5 pt] (a)

$\Rightarrow$

[5 pt] (b)

[5 pt] (c) $\int \frac{1}{v(v+3)^2}\, dv \;=\; \int \Big(\frac{A}{v}$

$1 = A(v+3)^2 + B$

$v = 0 \implies 1 = 9A$

$v = -3 \implies 1 = -3C$

$v = -2 \implies 1 = A -$

$\int \frac{1}{v(v+3)^2}\, dv = \int \Big($

$= \frac{1}{9} \ln$

$= \ln$

[5 pt] (d) $\int e^{\sqrt{3x+1}}\, dx$

$= \int e^{t} \cdot \frac{2}{3} t\, dt$

$= \frac{2}{3} \int t e^{t}\, dt$

By parts ——

$= \frac{2}{3}\left(t e^{t} - \int e^{t}\, dt\right)$

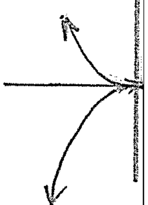
$= \frac{2}{3} t e^{t} - \frac{2}{3} e^{t} + C$

$= \frac{2}{3} \sqrt{3x+1}\, e^{\sqrt{3x+1}} - \frac{2}{3}$

[5 pt] 3. Fi

[5 pt] 4. Fi

[5 pt] 5. Deter



[5 pt] 6. Find tl

y'

L

[8 pt] 7. Let $\mathcal{R}$ be

(a) Stat
$\mathcal{R}$ al

(b) Stat
$\mathcal{R}$ al

(a)

$y=1$

(a)

(b)

$2-y$

8. In all

[2 pt] (a) I
(

[2 pt] (b) I

[1 pt] (c) If

(a

(b

(c

[3 pt] 9. Suppo

$\int_0^0$

$\lim_{x \to 0^+}$

] 10. Evalua

a

lim
n→∞

[5] 11. Suppos
integra

Dete

f(−x

∴

=

] 12. (a) Show that $0 < \frac{(}{(}$

] (b) Use part (a) (whe
or diverges. If it

(a) $\frac{(n!)^2}{(2n)!} >$

$\frac{(n!)^2}{(2n)!} =$

(b)

$\therefore a_n$

13. Determine if each of th

(a) $\pi - \frac{2\pi}{e} + \frac{4\pi}{e^2} - \frac{8}{e}$

Geometric

Since $|r|$

$S = \frac{a}{1-r}$

(b) $\sum_{n=1}^{\infty} a_n$ where the

$\sum_{n=1}^{\infty} a_n = \lim_{n\to}$

$\therefore$ the

14. Determine if each of the fol
gent. Clearly state which t
your conclusion.

[4 pt] (a) $\sum_{n=2}^{\infty} \frac{1}{n\sqrt[3]{\ln n}}$

Integral Tes

- $f(x) \geq 0$?
- $f(x)$ cts ?
- $f(x)$ dec

$\int_2^{\infty} \frac{1}{x\sqrt[3]{\ln x}}\,dx =$

$=$

$=$

$=$

Since $\int_2^{\infty} \frac{1}{x\sqrt[3]{\ln x}}$

[4 pt] (b) $\sum_{n=6}^{\infty}\left(1-\frac{5}{n}\right)^{n^2}$

Root test (no

$\lim_{n\to\infty} \sqrt[n]{\left|\left(1-\frac{5}{n}\right.\right.}$

$\Rightarrow \ln L = \lim_{n\to\infty}$

$= \lim_{n\to\infty}$

$= \lim_{n\to\infty}$

$= \lim_{n\to\infty}$

$= \lim_{n\to\infty}$

$= (1$

[4 pt] (c) $\sum_{n=2}^{\infty}(-1)^{n+1}\sqrt{\frac{1}{n^2-2}}$

Alternating serie

$a_n = (-1)^{n+1} \frac{}{\sqrt{n}}$

① $b_n \to 0$?

$\lim_{n\to\infty} \frac{1}{\sqrt{n^2-2}}$

② b_n decreasing

$\therefore \sum a_n$ conve

Now consider $\sum$

$\frac{1}{\sqrt{n^2-2}} >$

Since $\sum \frac{1}{n}$ d

Direct Compari

Since $\sum a_n$ co

$\sum a_n$ is condi

[4 pt] 15. (a) Find the interval c

[4 pt] (b) Show that the Tay

[1 pt] (c) Use (a) and (b) to

you found in (a).

(a) Ratio Test:

$\lim_{n\to\infty} \left| \frac{(-1)^{n+1} (}{(-1)^n (} \right.$

The series

At $x=2 \Rightarrow$

At $x=4 \Rightarrow$

Each of t

interval of

(b) $f^{(0)}(x) = ($

$f^{(1)}(x) = -2$

$f^{(2)}(x) = 2$

$f^{(3)}(x) = -$

$f^{(n)}(x) = ($

$f^{(n)}(3) = ($

$= ($

(c) $f(\frac{5}{2}) = \sum_{n=}^{\infty}$

$\frac{1}{(\frac{5}{2}-2)^2} = \sum_{n}$

$\frac{1}{(1/2)^2} = \sum_{n}$