

DAWSON COLLEGE
Mathematics Department
Final Examination
Linear Algebra
2016NYC605 (Commerce)
Fall 2019

1. a) (5 marks) Use Gauss-Jordan elimination to find the general solution of the system.

b) (1 mark) Find a particular solution in which $x_1 = 3$ and $x_3 = 5$.

$$\begin{array}{cccc} x_1 & x_2 & 2x_3 & x_4 & 7 \\ 2x_1 & x_2 & 2x_3 & & 8 \\ 3x_1 & 2x_2 & 4x_3 & x_4 & 15 \end{array}$$

2. (6 marks) Given the system of linear equations
$$\begin{array}{cccc} 2x & y & z & 1 \\ x & 2y & 2z & 1 \\ 3x & 2y & 5z & 5 \end{array}$$

a) Use the adjoint matrix to find the inverse of the coefficient matrix.

b) Use the inverse of the coefficient matrix to solve the system.

3. (3 marks) Use Cramer's rule to solve the system from question #2 hgt"özü"qpnf.

4. (3+1+4 marks) Let $A = \begin{pmatrix} 7 & 3 & 2 & 1 \\ 2 & 1 & 5 & 2 \\ & & 1 & 3 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ and $C = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$.

a. Calculate $tr(A^2 - 3B^T B + 4C^{2019})$

b. Does the system $B^T X = 0$ have a nontrivial solution? Justify your answer without solving the system.

c. Solve for X : $AX^T = 2I - A^2$

5. (3 marks) Determine the values of a such that the system has

1) a unique solution, 2) infinitely many solutions, 3) no solution :

$$\begin{array}{l} x - 2y + 4az = 3 \\ y + 7az = 2 \\ x + 3y - a^2 - 2a - z = a - 6 \end{array}$$

6. (3 marks) Let

a) $\begin{vmatrix} d & mg & 3a & g & g & a \\ e & mh & 3b & h & h & b \\ f & mi & 3c & i & i & c \end{vmatrix}$, where m is a real number.

b) $\det \begin{pmatrix} 2B^3 & 1 \\ B^T A & 1 \end{pmatrix}^4$

c) $\det \begin{pmatrix} A^{-1} B^{-1} & B^{-1} \\ B^{-1} & \text{adj}(A^{-1}) \end{pmatrix}$

10. (3+3+3+3 marks) Let $\vec{u} = 3, 2, 1$, $\vec{v} = 1, 2$, and $\vec{w} = \vec{i} + \vec{j} + 2\vec{k}$

a) Find the area of the triangle determined by \vec{u} , \vec{w} and $\vec{u} + \vec{v}$.

b) Find a vector of length 5 perpendicular to both vectors $\vec{u} + \vec{w}$ and $\vec{u} + \vec{v}$.

c) Find $\text{Proj}_{\vec{u} + \vec{v}} \vec{u} + \vec{w}$

d) Find the value(s) of k such that the vector $\vec{v} + k\vec{w}$ is perpendicular to $\vec{v} + k\vec{u}$.

11. (3 marks) Let $\vec{a} \cdot \vec{c} = \vec{a} \cdot \vec{b} = \vec{a} \cdot \vec{c} = 4$. Find the volume of the parallelepiped with edges $\vec{a}, \vec{b}, \vec{c}$.

12. (1+3 marks) a) Determine whether the planes $x + 2y + z = 2$ and $2x + 3y + 5z = 1$

Answers

1. a) $x_1 = 1 - t$, $x_2 = 6 - 2s - 2t$, $x_3 = s$, $x_4 = t$. b) $x_1 = 3$, $x_2 = 8$, $x_3 = 5$, $x_4 = 2$.

$$\frac{6}{7} \quad 1 \quad \frac{4}{7}$$

2. a) $A^{-1} = \begin{pmatrix} \frac{1}{7} & 1 & \frac{3}{7} \\ \frac{4}{7} & 1 & \frac{5}{7} \end{pmatrix}$; b) $x = 1$, $y = 1$, $z = 2$.

$$\frac{4}{7} \quad 1 \quad \frac{5}{7}$$

3. $z = 2$

4. a) 94; b) Yes, it does. c) $X = \begin{pmatrix} 29 & 70 \\ 105 & 251 \end{pmatrix}$.

5. 1) $a = 0$, $a = 1$; 2) $a = 1$; 3) $a = 0$

6. $A^{-1} = \begin{pmatrix} 1 & 2 & 1 & 0 \\ 0 & 1 & 0 & \frac{1}{3} \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}$. Other possible answers.

7. $BC = CB$

8. 80.

9. a) 8; b) $\frac{1}{32}$; c) $\frac{1}{16}$

10. a) $\sqrt{19}$; b) $\frac{15}{\sqrt{19}}$, $\frac{5}{\sqrt{19}}$, $\frac{15}{\sqrt{19}}$; c) $\frac{5}{2}, 0, \frac{5}{2}$; d) $k = 2$, $k = 3$.

11. 2

12. a) not parallel; b) .

13. a) $\frac{\sqrt{910}}{\sqrt{61}}$; b) $15x - 26y - 3z - 71 = 0$; c) 0,1,7; d) 1, 1,10 .

14. $2\sqrt{5}$

15. True;

16. $P = 27$, $x_1 = 0$, $x_2 = 5$, $x_3 = 7$.

17. $C = 14$, $x_1 = 7$, $x_2 = 0$.