

DAWSON COLLEGE
Mathematics Department
Final Examination
Linear Algebra
2016NYC05 (Commerce)
Fall 2021

1. a) (5 marks) Use Gauss-Jordan elimination to find the general solution of the system.
 b) (1 mark) Find a particular solution in which $x_2 = 5$ and $x_3 = 0$.

$$\begin{array}{rcl} x_1 & 2x_2 & 2x_3 & 4x_4 & 3 \\ 3x_1 & 6x_2 & 5x_3 & 14x_4 & 1 \\ 4x_1 & 8x_2 & 6x_3 & 20x_4 & 4 \end{array}$$

$$2x + y + 3z = 3$$

2. (6+4 marks) Given the system of linear equations $x + 4y + z = 6$.

$$2x + 4y + z = 11$$

- a) Solve the system using the inverse matrix. Use the **adjoint** matrix to find the inverse.

- b) Use A^{-1} to solve the system $AX = B$.

$$\begin{bmatrix} 1 & 3 \\ 2 & 1 \\ 0 & 2 \end{bmatrix} X = \begin{bmatrix} 2 \\ 4 \\ 1 \end{bmatrix}$$

3. (4 marks) Let $A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \\ 0 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 1 \\ 4 & 2 \\ 1 & 3 \end{bmatrix}$, $C = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$. Solve for X : $CX^{-1} = I^{-1} = XA^T B$.

4. (3 marks) If $A = \begin{bmatrix} 4 & 2 \\ 1 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 1 \\ 7 & 7 \end{bmatrix}$, find elementary matrices E_1 and E_2 such that $E_2 E_1 A = B$.

5. (3 marks) Simplify as much as possible $B^{-1} - 2C^{-1} B^{-1} + D^T C^{-1} + 4A^T D^{-T} + I + BD^0 + 2A^{-1}$.

6. (4 marks) Determine the values of a such that the system has
 1) a unique solution, 2) infinitely many solutions, 3) no solution:

$$x + 2y + 4z = 3$$

$$y + 7az = 2$$

$$x + 3y + a^2 z = a + 6$$

7. (3 marks) Let A be an invertible

16. (7 marks) Maximize $P = 2x_1 + x_2 + 4x_3$ subject to

$$\begin{cases} 4x_1 + 2x_2 + 3x_3 = 3 \\ 3x_1 + 2x_2 + 3x_3 = 15 \end{cases}$$

17. (7 marks) Minimize $C = 2x_1 + 9x_2$ subject to

$$\begin{cases} 2x_1 + 5x_2 = 2 \\ x_1 + 3x_2 = 5 \\ 3x_1 + x_2 = 1 \\ x_1, x_2 \geq 0 \end{cases}$$

Answers

1. a) $x_1 = 13 - 2t - 8s$, $x_2 = t$, $x_3 = 8 - 2s$, $x_4 = s$. b) $x_1 = 9$, $x_2 = 5$, $x_3 = 0$, $x_4 = 4$.

2. a) $A^{-1} = \begin{pmatrix} \frac{8}{19} & \frac{13}{19} & \frac{11}{19} \\ \frac{1}{19} & \frac{4}{19} & \frac{1}{19} \\ \frac{12}{19} & \frac{10}{19} & \frac{7}{19} \end{pmatrix}$, $x = 1$, $y = 2$, $z = 1$.; b) $y = 2$

3. $X = A^T B^{-1} C = \begin{pmatrix} \frac{3}{2} & \frac{3}{2} \\ \frac{11}{5} & \frac{23}{5} \end{pmatrix}$

4. $E_1 = \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & 1 \end{pmatrix}$, $E_2 = \begin{pmatrix} 1 & 0 \\ 4 & 1 \end{pmatrix}$. *Other possible answers.*

5. I

6. 1) $a = 1$, $a = 6$; 2) $a = 1$; 3) $a = 6$

7. skew-symmetric

8. -57

9. a) 24; b) $\frac{729}{2}$; c) 32

10. a) ; b) $\frac{12}{7}$, $\frac{4}{7}$, $\frac{8}{7}$; c) 1.9.

11. a) $\frac{\sqrt{14}}{2}$; b) $\frac{1}{\sqrt{14}}$, $\frac{2}{\sqrt{14}}$, $\frac{3}{\sqrt{14}}$, $\sqrt{\quad}$

12. a) $x = 4 - 3t$, $y = 5 - 5t$, $z = t$; b) $x = 2 - 3s$, $y = 1 - 5s$, $z = 3 - s$.

13. a) $\sqrt{5}$; b) 0, 1, 4

14. True

15. b) $\sqrt{\quad}$; c) $y \ z \ 4 \ 0$

16. $P \ 13, x_1 \ 0, x_2 \ 5, x_3 \ 2.$

17. $C \ 10,$
;