

9. (12 marks) A and B are 3×3 matrices and $\det A = \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = 4$, and $\det B = 3$. Find

a) $\det A^T (2B)^3 \det(B)$ b) $\det 5BA^{-1} - 2\text{Badj}(A)$

c) $\begin{vmatrix} 2a & 3g & 5g & 4a & d \\ 2b & 3h & 5h & 4b & e \\ 2c & 3i & 5i & 4c & f \end{vmatrix}$

10. (4 marks) Evaluate the determinant $\begin{vmatrix} 2 & 1 & 5 & 1 \\ 8 & 0 & 1 & 3 \\ 1 & 1 & 6 & 2 \\ 3 & 1 & 5 & 3 \end{vmatrix}$ by row reduction. **You must perform at least**

one row operation.

11. (9 marks) Let $\vec{u} = \langle 2, 1, 3 \rangle$, $\vec{v} = \langle 1, 2, 4 \rangle$, $\vec{w} = \langle 3, 1, \dots \rangle$.

- Find the orthogonal projection of the vector \vec{w} on the vector $\vec{u} - \vec{v}$, that is $\text{Proj}_{\vec{u} - \vec{v}} \vec{w}$.
- Find a unit vector perpendicular to $\vec{u} - \vec{v}$ and \vec{w} .
- Find the area of a triangle determined by $\vec{u} - \vec{v}$ and \vec{w} .

partial (marks)

41. (marks) fy

Answers

1. a) $x_1 = 2 - 3t - s$, $x_2 = 1 - t + 5s$, $x_3 = t$, $x_4 = s$. b) $x_1 = 1$, $x_2 = 4$, $x_3 = 0$, $x_4 = 1$.

$$1 \quad \frac{1}{3} \quad \frac{5}{3}$$

2. a) $A^{-1} = \begin{pmatrix} 1 & 0 & 1 \\ 1 & \frac{2}{3} & \frac{4}{3} \end{pmatrix}$; b) $x = 1$, $y = 2$, $z = 1$.

$$1 \quad \frac{2}{3} \quad \frac{4}{3}$$

3. $y = 2$

4. $4A^2$

5. a) -47 ; b) $X = \begin{pmatrix} 3 & 4 \\ 2 & 3 \end{pmatrix}$

6. $E_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, $E_2 = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}$. *Other possible answers.*

7. 1) $k = 3$; 2) $k = 3$; 3) $k = 3$

8. False.

9. a) $\frac{1}{128}$ b) $\frac{81}{4}$; c) 40

10. -16

11. a) $\frac{51}{19}$, $\frac{51}{19}$, $\frac{17}{19}$; b) $\frac{7}{\sqrt{94}}$, $\frac{6}{\sqrt{94}}$, $\frac{3}{\sqrt{94}}$; c) $\sqrt{94}$.

12. a) Yes, the line is parallel to the plane; b) ; c) $2\sqrt{14}$.

13. $x = 11 - 13t$, $y = 3 + 2t$, $z = t$.

14. 0

15. b) $29x - 3y - 13z - 118 = 0$

16. $P = 43$, $x_1 = 3$, $x_2 = 0$, $x_3 = 4$, $x_4 = 0$.

17. $C = 60$, $x_1 = 0$, $x_2 = 20$, $x_3 = 0$.