Dawson College Department of Mathematics – Winter 2011

Instructors : I. Rajput, A. Jimenez, M. Marchant, M. Ishii

Final Examination Tuesday May 17th 2011
Calculus I (201-103-DW)

STUDENT NAME: ______________________________

[redacted]

No graphing / programmable calculators allowed.
There are 16 questions in total worth 100 marks.
Show all your work in the space provided and circle your final answers.

[redacted]

(a) $\lim\limits_{x\to 2} \dfrac{3x^2+6x-24}{x^2-4} = \dfrac{0}{0}$

$= \lim\limits_{x\to 2} \dfrac{3(x-2)(x+4)}{(x-2)(x+2)}$

$= \lim\limits_{x\to 2} \dfrac{3(x+4)}{x+2} = \dfrac{9}{2}$

(b) $\lim \dfrac{x}{\quad} = \dfrac{0}{\quad}$

[redacted]

$= \lim\limits_{x\to 0} \dfrac{x}{\sqrt{1-3x}-1} \cdot \dfrac{(\sqrt{1-3x}+1)}{(\sqrt{1-3x}+1)}$

[3] **2. Consider the following piece-wise defined function:**

$$f(x) = \begin{cases} \sqrt[3]{2+x-3x^2} & \text{if } x \geq 2 \\ 3x-8 & \text{if } x < 2 \end{cases}$$

Find the limits:

(a) $\lim_{x\to 2^+} f(x) = \sqrt[3]{2+2-12} = -2$

(b) $\lim_{x\to 2^-} f(x) = 6-8 = -2$

$\begin{cases} x^2 - x - 12 \end{cases}$ if ... 3

State the definition for a function to be continuous at a point.

definition to evaluate $f'(x)$ for $f(x)=\frac{3}{2}x^2+7x-1$.

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$\lim \quad \frac{3}{2}(x+h)^2+7(x+h)-1-\left(\frac{3}{2}x^2+7x-1\right)$$

[6] 5 Find the equation of the tangent line to the graph of $f(x) = x$

at the point where $x = 1$.

$$m = f'(1)$$

$$f'(x) = \frac{(7-3x)^{1/2}(1) - x\,\frac{1}{2}(7-3x)^{-1/2}(-3)}{(7-3x)}$$

$$= f'(1) = \frac{11}{16} \rightarrow y = \frac{11}{16}x + b$$

$$= f(1) = \frac{1}{2}$$

$$\frac{1}{2} = \frac{11}{16} + b$$

$$b = \frac{-3}{16}$$

$$y = \frac{11}{16}x - 3$$

[5] **6. For the function $f(x)=\dfrac{2x+1}{}$ find $f''(x)$ and simplify your answer.**

$$= -3(x-1)^{-2}$$

$$f''(x) = 6(x-1)^{-3}$$

$$= \frac{6}{(x-1)^3}$$

[5] **7. Compute the value of $f'(x)$ when $x=0$ given that**
$f(x)=(e^{-2x}+3)^3+\sin(6x)$.

$$f'(x) = 3(e^{-2x}+3)^2(e^{-2x})(-2) + 6\cos(6x)$$

[7] **8. Suppose the monthly revenue and cost functions (in dollars) for x units of a commodity sold and produced are** $R(x) = 400x - \frac{x^2}{20}$ **and** $C(x) = 5000 + 70x$.

a) Find the Profit function $P(x)$ **.**

$$P(x) = R(x) - C(x)$$

$$= -\frac{x^2}{20} + 330x - 5000$$

b) Find the marginal profit function and use it to estimate the profit from selling the 42nd unit.

$$P'(x) = -\frac{2x}{20} + 330 = -\frac{x}{10} + 330$$

$$P'(41) = -\frac{41}{10} + 330 = 325.90 \ \$/\text{unit}$$

c) Calculate the actual profit from selling the 42nd unit.

$$P(42) - P(41) = 325.85 \ \$$$

[5] ____ 9. Find the x-value(s) where the graph of $f(x) = \frac{3x-2}{\quad}$ has a horizontal

$$f'(x) = \frac{(2x+1)^{1/2}\,3 - (3x-2)\tfrac{1}{2}(2x+1)^{-1/2}\cdot 2}{(2x+1)}$$

$$= \frac{(2x+1)^{-1/2}\left[(2x+1)3 - (3x-2)\right]}{(2x+1)}$$

$$= \frac{6x+3 \quad -3x+2}{(2x+1)^{3/2}}$$

$$= \frac{3x+5}{(2x+1)^{3/2}} = 0$$

[6] **10. Consider the relation** $\sqrt{x+y} = 1 + x^2y^2$.

(a) Find the derivative y'.

$$\frac{1}{2}(x+y)^{-1/2}(1+y') = x^2 2yy' + 2xy^2$$

$$(x+y)^{-1/2} + (x+y)^{-1/2}y' = 4x^2yy' + 4xy^2$$

$$(x+y)^{-1/2}y' - 4x^2yy' = -(x+y)^{-1/2} + 4xy^2$$

$$y'\left[(x+y)^{-1/2} - 4x^2y\right] = -(x+y)^{-1/2} + 4xy^2$$

$$y' = \frac{-(x+y)^{-1/2} + 4xy^2}{(x+y)^{-1/2} - 4x^2y} = \frac{4xy^2\sqrt{x+y} - 1}{1 - 4x^2y\sqrt{x+y}}$$

(b) Compute the value of y' **at** $(0,1)$.

$$y' = -1$$

[redacted]

$$f(x) = (\cos 2x)^{\ln x}.$$

$$y = (\cos 2x)^{\ln x}$$

$$\ln y = \ln (\cos 2x)^{\ln x}$$

$$\ln y = (\ln x) \cdot \ln(\cos 2x)$$

$$\frac{y'}{y} = \ln x \, \frac{(-\sin 2x)\cdot 2}{\cos 2x} + \ln(\cos 2x) \cdot \frac{1}{x}$$

[5] **12. Consider the function** $f(x) = \ln\left(\frac{e^x\sqrt{x^2+1}}{(x-2)^3}\right)$.

(a) Fully simplify $f(x)$ **using the properties of logarithms.**

$f(x) = \ln(e^x) + \frac{1}{2}\ln(x^2+1) - 3\ln(x-2)$

$= x + \frac{1}{2}\ln(x^2+1) - 3\ln(x-2)$

(b) Find $f'(x)$ **and simplify your answer.**

$f'(x) = 1 + \frac{1}{2}\frac{2x}{(x^2+1)} - 3\frac{(1)}{x-2}$

$= 1 + \frac{x}{x^2+1} - \frac{3}{x-2}$

[12] **13. Consider the function** $f(x) = x^3 - 6x$

(a) Find the x and y intercepts.

y-int: $y = 0$

x-int: $x(x^2 - 6) = 0$

$x = 0$ $\quad$ $x = \pm\sqrt{6}$

$= \pm 2.45$

(0, 0)
(2.45, 0)
(−2.45, 0)

(b) Find the intervals of increase/decrease and the relative extrema.

$f'(x) = 3x^2 - 6 = 0$

$3(x^2 - 2) = 0$

$x = \pm\sqrt{2} = \pm 1.41$

Inc. Dec. Inc.

f': $+$ | $-$ | $+$

$-\sqrt{2}$ $\quad$ $\sqrt{2}$

max $\quad$ min

local max (−1.41, 5.66)

local min (1.41, −5.66)

(c) Find the intervals of concavity and the points of inflection.

$f''(x) = 6x = 0$

$x = 0$

Down Up

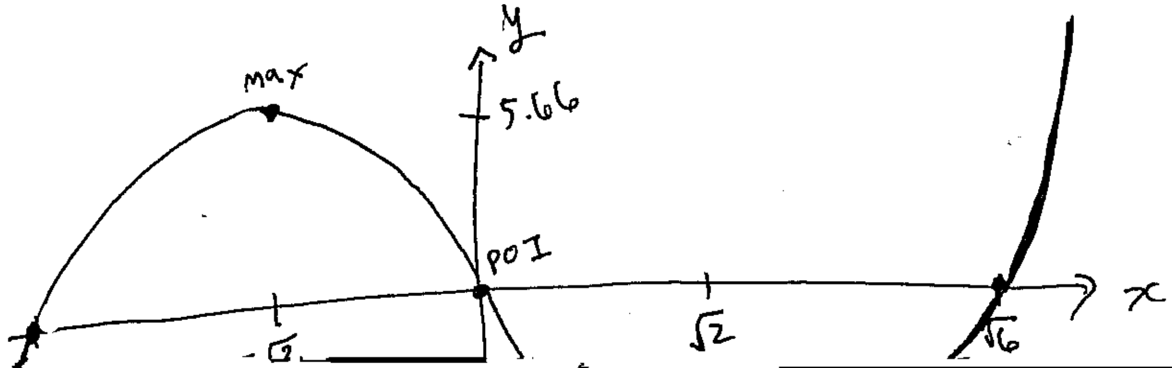
f'': $-$ | $+$

0

POI

POI (0, 0)

(d) Sketch a graph of $f(x)$ with important points labeled.



[8] **16. Find the antiderivatives:**

(a) $\int \sqrt{x}(2x^3 - 7)\, dx$

$\int (2x^{7/2} - 7x^{1/2})\, dx$

$= \frac{4}{9} x^{9/2} - \frac{14}{3} x^{3/2} + C$

(b) $\int \frac{5x-1}{\sqrt[3]{5x^2 - 2x + 1}}\, dx$

$u = 5x^2 - 2x + 1$

$du = 10x - 2\, dx$

$du = 2(5x-1)\, dx$

$\frac{du}{2} = (5x-1)\, dx$

$\int \frac{du}{2u^{1/3}}$