

Dawson College
Mathematics Department
Final Examination

701 DZE 05 01 2010

Monday December 20th, 2010

Student Name: _____

Student I.D. #: _____

Instructor: Richard Fournier

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INSTRUCTIONS:

- Print your name and student I.D. number in the space provided above.
- Answer 15 questions out of 20 (7 marks/question).
- All questions are to be answered directly on the examination paper.
- Translation and regular dictionaries are permitted.
- Small, noiseless, NON-PROGRAMMABLE calculators without text storage or graphic capabilities are permitted.
- Please ensure that you have a complete exam package before starting.

The exam must be returned intact.

<i>Question #</i>	<i>Marks</i>
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1. [7 marks] Parametrize the curve with respect to arc length:

$$r(t) = (e^t \cos(2t), e^t \sin(2t), 2 + \pi), 0 \leq t \leq \pi.$$

$$t = \ln\left(\frac{R}{\sqrt{5}} + 1\right), \quad 0 \leq R \leq \sqrt{5}(e^\pi - 1)$$

2. [7 marks] Find the angle between T and T' at $t=1$ if $r(t) = (t, t^2, t^3)$.

$$0 = \frac{d}{dt} |T|^2 = \frac{d}{dt} T \cdot T = 2 T \cdot T' \Rightarrow \angle(T, T') = 90^\circ$$

$\approx \frac{\pi}{2} \text{ rad.}$

3. [7 marks] Compute $\lim_{x \rightarrow 0} \frac{xy \sin(x+y)}{x^2 + y^2}$.

0

4. [7 marks] Find the equation of the tangent plane to the surface $x^4 + y^4 + z^4 = 2$ at $(1, 1, 0)$.

$$x + y = 2$$

7. [7 marks] Prove that $\left(\frac{x^2+y^2+z^2}{3}\right)^2 \leq \frac{x^4+y^4+z^4}{3}$ for x, y, z real.

HINT: Maximize $x^2+y^2+z^2$ under the constraint $x^4+y^4+z^4=1$.

Call $f(x, y, z) = x^2 + y^2 + z^2$ and $g(x, y, z) = x^4 + y^4 + z^4 - 1$
According to Lagrange, $\exists L: \nabla f + L \nabla g = (0, 0, 0)$
Max = $\sqrt{3}$

8. [7 marks] Find all critical points of $f(x, y) = (1 - x^2 - y^2)^2$ and classify them.

at $(0, 0)$: local max
at any (x, y) with $x^2 + y^2 = 1$: local min

9. [7 marks] Assuming that the function $f(x, y) = \sqrt{xy}$ is differentiable at $(1, 1)$, find an approximation to $\sqrt{(1.001)(1.0001)}$.

$$\frac{\partial^2 f}{\partial x^2} \quad \frac{\partial^2 f}{\partial y^2} \quad \dots \quad e^y + e^{-y} \quad \dots \quad e^y - e^{-y}$$

13. [7 marks] Compute $\left(\iint_R (x^2 + y^2) dA \right)^2$ where $R = [0,1] \times [0,1]$. $\frac{4}{9}$

14. [7 marks] Compute $\iint_R e^{x^2+y^2} dA$ where $R = \{(x,y) \mid 0 \leq x^2 + y^2 \leq 1\}$. $\pi(2-1)$

15. [7 marks] Compute

$\iiint e^{x^2+y^2} dV$ where R is the cylinder $\{(x,y,z) \mid x^2 + y^2 \leq 1 \text{ and } 0 \leq z \leq 1\}$.

16. [7 marks] Compute

$\iiint_R dV$ where R is the sphere $\{(x,y,z) \mid x^2 + (y-1)^2 + (z-2)^2 \leq 2\}$.
 $\frac{8\sqrt{2}\pi}{3}$

17. [7 marks] Find a power series representation for $f(x) = \ln(x+1)$.

18. [7 marks] Test for convergence: $\sum_{n=1}^{\infty} (-1)^{n-1} (n^{1/n} - 1)$.

$n^{1/n}$ decreases to 1 as n increases!

The series converges (Leibniz!)

19. [7 marks] Find the interval of convergence for $\sum_{n=2}^{\infty} \frac{2^n}{n^2} (x-2)^{4n}$.

$$\left[2 - \frac{1}{\sqrt[4]{2}}, 2 + \frac{1}{\sqrt[4]{2}} \right]$$

20. [7 marks] Find the sum of the series $\sum_{n=1}^{\infty} nx^{2n}$. $\frac{x^2}{(1-x^2)^2}$, $|x| < 1$