

$$(b) \int x(x^2 + 3)^4 dx$$

16. (16 marks) Consider the function $f(x) = \frac{x^3 - 1}{x^3 + 1}$. The first and second derivatives of f are

$$f'(x) = \frac{6x^2}{(x^3 + 1)^2} \quad \text{and} \quad f''(x) = \frac{12x(1 - 2x^3)}{(x^3 + 1)^3}.$$

- Give the domain of f .
- Find the intercepts.
- Find the horizontal and/or vertical asymptotes (if any).
- Find the intervals where f is increasing, and the intervals where f is decreasing.
- Find the local maxima and local minima.
- Find the intervals where f is concave up, and the intervals where f is concave down. Give the inflection point(s).
- Use the information above to sketch the graph of f . Clearly label all the important points on the graph.

Answers:

- (a) 1=4 (b) 2 (c) 3 (d) 1
- 4
- $\frac{1}{(x+1)^2}$
- $f'(x) = \frac{\sin x}{2x} + \frac{\cos x}{x}$
- $x = 2$ and $x = -1 = 2$
- $f''(x) = 4 - 4 \sin(2x)$
- $\frac{\sin x}{6xy} - \frac{3y^2}{1} - \frac{\sec^2 x}{1}$
- 3=5
- $y' = x^{\sin x} (\cos x \ln x + \frac{\sin x}{x})$
- $y = 2x + 3$
- $\frac{2}{(x^2 + 1)^{3/2}}$
- 10:4 feet per second
- 4000 cm³
- Absolute maximum value is 1, absolute minimum value is -5.
- (a) $\frac{2x^{2/3}}{3} + \frac{2x^{1/2}}{3} + \frac{x^3}{3} + C$ (b) $\frac{(x^2 + 3)^5}{10} + C$
- (a) $\mathbb{R} \setminus \{1\}$ (b) x -intercept (1;0), y -intercept (0; -1) (c) vertical asymptote: $x = 1$. 40005000