**Question 1.** If the limit exists, find its value; otherwise explain why the limit doesn't exist.

a. *(3 marks)*

$$\lim_{x \to 9} \frac{3}{x-3}$$

Answer: DNE

b. *(3 marks)*

$$\lim_{x \to \infty} \frac{3x^3 - x + 1}{2 - 5x^3}$$

Answer: $\frac{3}{5}$

c. *(3 marks)*

$$\lim_{x \to 2} \frac{x^2 - 4}{x^4 + 2x^3}$$

Answer: $\frac{1}{2}$

**Question 2.** *(5 marks)* For which values of $x$ is the following function continuous? Clearly explain your reasoning.

$$f(x) = \begin{cases} \frac{3x-2}{x^2-1} & \text{if } x < 2 \\ 2 & \text{if } x = 2 \\ x^4 + x^3 + x + 2 & \text{if } x > 2 \end{cases}$$

Answer: $R \, nf \, 2g$
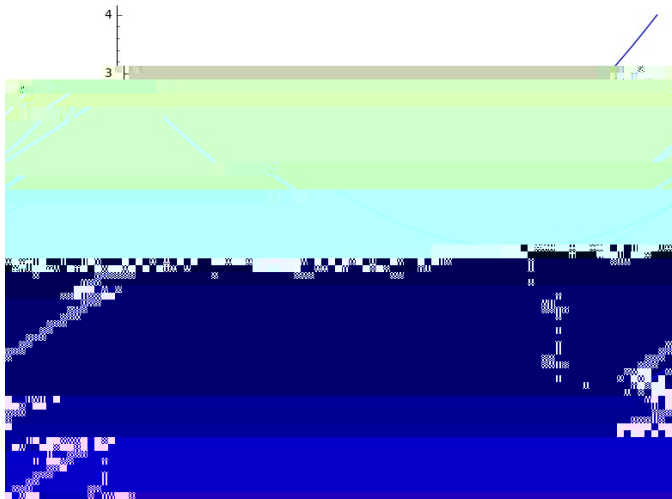
**Question 3.** Given

$$f(x) = x^2 - 2x + 1$$

a. *(4 marks)* Find the derivative of $f(x)$ using the definition of the derivative as a limit.

Answer: $f'(x) = 2x - 2$

b. *(3 marks)* Find the tangent to $f(x)$ at $x = 2$.

Answer: $y = 2x - 3$

c. *(3 marks)* Sketch the graph of $f(x)$ and its tangent at $x = 2$.

**Question 4.** (4 marks) Find the absolute maximum value and the absolute minimum value of the given function

$$f(x) = \frac{x}{x^2 + 1} \quad \text{on} \quad [\ 1, 1]:$$

**Answer:** abs. min. $f(\ 1) = \frac{1}{2}$, abs. max. $f(1) = \frac{1}{2}$

**Question 5.** Find the derivative of the following functions:

a. (5 marks)

$$f(x) = \arcsin e^{2x}$$

**Answer:** $f'(x) = \frac{2e^{2x}}{1\ e^{4x}}$

b. (5 marks)

$$f(x) = \frac{(\ln x + x^2)^2}{\sin x}$$

**Answer:** $f'(x) = \frac{(\ln x + x^2)\left[\left(\frac{2}{x} + 4x\right)\sin x\ (\ln x + x^2)\cos x\right]}{\sin^2 x}$

c. (5 marks)

$$f(x) = (x^2 + 1)\arctan x$$

**Answer:** $f'(x) = 1 + 2x \arctan x$

**Question 6.** (5 marks) Using logarithmic differentiation, find the derivative of the function

$$f(x) = (x^3 + 1)^{12} \sqrt[3]{2x^2 + 5x}(x\tan x)^3:$$

Do not simplify (expand) your answer.

**Answer:** $f'(x) = (x^3 + 1)^{12}\sqrt[3]{2x^2 + 5x}(x\tan x)^3 \left[ \frac{36x^2}{x^3+1} + \frac{4x+5}{6x^2+15x} + \frac{3}{x} + \frac{2\sec^3 x}{\tan x}\right]$

**Question 7.** (5 marks) Find an equation of the tangent line to the curve $x^2 y^3\ y^2 + xy\ 1 = 0$ at the point $(1, 1)$.

**Answer:** $y = \frac{3}{2}x + \frac{5}{2}$

**Question 8.** (5 marks)

a. *(3 marks)* Show that the demand equation is $p = \ 0.0000002x^3 + 6$ and show that the monthly revenue (in dollars) for selling $x$ copies of the book is $R(x) = \ 0.0000002x^4 + 6x$.

b. *(2 marks)* Compute $R'(100)$ and give an interpretation meaningful to someone who does not know Calculus.

**Answer:** $R'(100) = 5.2$, the approx. revenue generated by selling the $101^{st}$ book is 5.20$

c. *(3 marks)* Compute the marginal averate cost $\bar{C}'(x)$.

**Answer:** $\bar{C}'(x) = \ \frac{200}{x^2} \ \ 0.0006x$

**Question 10.** *(6 marks)* Find the third derivative of the following function. Simplify your final answer.

$$f(x) = \frac{2x \ 1}{(3x+1)^3}$$

**Answer:** $f'(x) = \frac{12x+11}{(3x+1)^4}$ ; $f''(x) = \frac{36[3x \ 4]}{(3x+1)^5}$ ; $f'''(x) = \frac{324[\ 4x+7]}{(3x+1)^6}$

**Question 11.** Consider the function

$$f(x) = 4x^5 + 5x^4$$

a. *(1 mark)* State the domain of $f(x)$.

**Answer:** $\mathbb{R}$

b. *(3 marks)* Find the x-intercepts (if any) and the $y$-intercept.

**Answer:** $(0,0)$ ; $\ \frac{5}{4}, 0$

c. *(3 marks)* Find the intervals where $f(x)$ is increasing, and the intervals where $f(x)$ is decreasing.

**Answer:** $f(x)$ increasing on: $( \ \ Y; \ \ 1)$ ; $(0, Y)$, $f(x)$ decreasing on: $( \ \ 1, 0)$

d. *(1 mark)* Find the local maxima and local minima.

**Answer:** local min. at $(0, 0)$, local max. at $( \ \ 1, 1)$

e. *(3 marks)* Find the intervals where $f(x)$ is concave up, and the intervals where $f(x)$ is concave down. Give the inflection point(s).

**Answer:** $f(x)$ concave up on: $\ \frac{3}{4}, 0$ ; $(0, Y)$, $f(x)$ concave down on: $\ \ Y; \ \frac{3}{4}$ , inflection point at $\ \frac{3}{4}, \frac{81}{128}$

f. *(4 marks)* Use the information above to sketch the graph of $f(x)$ *(your graph has to agree with the previous answers)*
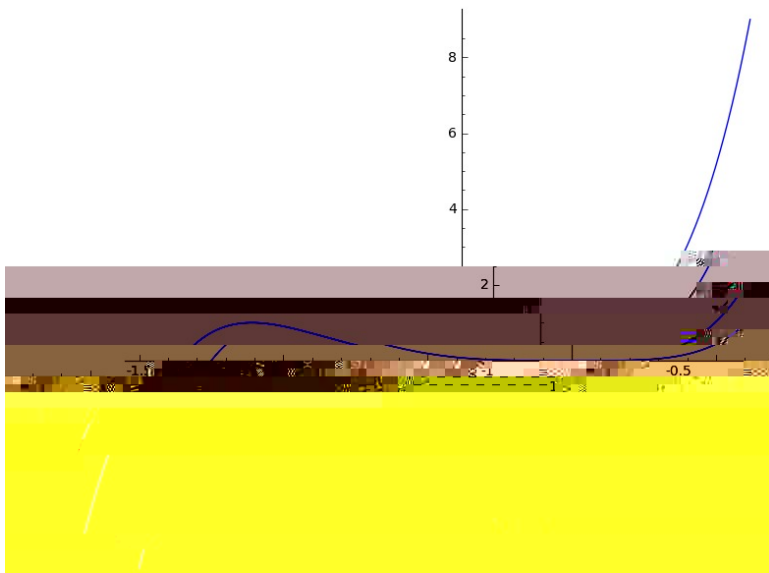
**Question 12.** *(4 marks)* Find the horizontal asymptote(s) and vertical asymptote(s) (if any) of the following function.

$$f(x) = \frac{x^2 - 2x}{x^2 - 1}$$

**Answer:** $x = 1; x = -1; y = 1$

**Question 13.** *(4 marks)* Evaluate the following integral.

$$\int$$