

Dawson College, Mathematics Department
Final Examination

Linear Algebra 201-NYC-05 (Commerce)
(Sections 6, 7)

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1. (6 marks) Solve the linear system by Gauss-Jordan method.

$$\begin{array}{cccc|c} x_1 & x_2 & 3x_3 & 4x_4 & 12 \\ 2x_1 & x_2 & 5x_3 & 7x_4 & 19 \\ x_1 & x_2 & 2x_3 & 2x_4 & 5 \end{array}$$

Ans. $x_1 = 1 + 3t$, $x_2 = 2 - t$, $x_3 = 3$, $x_4 = t$

2. (6 marks) For which values of k the system has
a) Exactly one solution, b) No solution, c) Infinitely many solutions

$$\begin{array}{ccc|c} x & 2y & z & 3 \\ 2x & 5y & 3z & 8 \\ x & 4y & k & 3z & k & 8 \end{array}$$

Ans. a) $k \neq 0$ b) $k = 0$ c) never

3. (5 marks) Let A be an invertible matrix, such that $2A^3 - 3A^2 - I = 2A$. Show that
 $A^{-1} = 2A^2 - 3A - 2I$

Ans. $AA^{-1} = A(2A^2 - 3A - 2I) = 2A^3 - 3A^2 - 2A = I$
 $A^{-1}A = (2A^2 - 3A - 2I)A = 2A^3 - 3A^2 - 2A = I$

4. The following linear system is given.

$$\begin{array}{rcl} x & 2y & z & 1 \\ 2x & y & z & 1 \\ x & y & z & 2 \end{array}$$

- a) (5 marks) Find the inverse of the coefficient matrix
 b) (3 marks) Use the result found in part a) to solve the system

Ans. a) $\begin{pmatrix} 0 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & 5 \end{pmatrix}$ b) $x = 1, y = 3, z = 6$

5. (5 marks) Use the Cramer's rule to solve the following system for y only.

$$\begin{array}{rcl} x & 2y & z & 1 \\ 2x & y & z & 1 \\ x & y & z & 2 \end{array}$$

Ans. $y = 3$

6. (5 marks) Find the matrix X , if

$$\begin{pmatrix} 1 & 2 \\ 3 & 5 \end{pmatrix} X \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 2 \\ 1 & 1 \end{pmatrix}$$

Ans. $\begin{pmatrix} 12 & 3 \\ 7 & 2 \end{pmatrix}$

7. Let A and B be 2×2 matrix, such that $\det A = 2$, $\det B = 3$. Find

- a) (4 marks) $\det A^{-1}B^T$
 b) (4 marks) $\det 2|A|B^{-1}$
 c) (4 marks) $\det B \operatorname{adj} 3A$

d) (5 marks) Let $\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = 5$. Find $\begin{vmatrix} d & g & a \\ 3e & d & 3h & g & 3b & a \\ 2f & d & 2i & g & 2c & a \end{vmatrix}$

Ans. a) $\frac{3}{2}$ b) $\frac{1}{3}$ c) 54 d) 30

8. (3 marks) Let \hat{u} and \hat{v} be two vectors, such that $\hat{u} \cdot \hat{v} = 1, 2, 3$. Find the vector

$\hat{u} \cdot \hat{v} - 2\hat{v} \cdot \hat{u}$ Ans. 3, 6, 9

9. (3 marks) Let $\vec{u} = k, 1, k, 2$, $\vec{v} = 2, 1, k$. Find the values of k for which $|\vec{u} \times \vec{v}| = 4$.

Ans. $k = 0, k = \frac{3}{2}$

10. Let $\vec{u} = 1, 1, 0$ and $\vec{v} = 1, 0, 2$

a) (3 marks) Find $\text{Proj}_{\vec{u}} \vec{v}$

b) (3 marks) Find the area of the triangle determined by \vec{u} and \vec{v}

Ans. a) $\frac{3}{2}, \frac{3}{2}, 0$ b) $\frac{3}{2}$

11. (4 marks) Find the equation of the plane which is perpendicular to the plane $x + y + 2z = 3$ and passes through the points $A(1, 0, 2)$ and $B(0, 1, -1)$.

Ans. $x + y - 1 = 0$

12. (4 marks) Find the point on the line $x = 6 - t, y = 2 + t, z = 3 - t$ which is the closest to the point $B(0, 1, 1)$

Ans. $x = 3, y = 1, z = 0$

13) (4 marks) Find the distance from the point $A(1, 0, 1)$ to the line $x = 1 + t, y = 2 + t, z = 3 - t$

Ans. $\frac{2}{3}\sqrt{6}$

14. (4 marks) Find the distance between the lines $\begin{matrix} x = 1 + 2t \\ y = 2 + t \\ z = 3 - t \end{matrix}$ and $\begin{matrix} x = 1 + s \\ y = s \\ z = 1 + s \end{matrix}$

Ans. $\sqrt{2}$

15. (10 marks) Maximize $P = 4x_1 + 3x_2 + 2x_3$ subject to $\begin{matrix} 2x_1 + 5x_2 + x_3 = 40 \\ 2x_1 + 2x_2 + x_3 = 15 \\ x_1 + 2x_2 + x_3 = 2 \end{matrix}$

Ans. Max $P = 30$ at $(13, 0, 11)$

16. (10 marks) Minimize $C = 3x_1 + 2x_2 + 2x_3$ subject to $\begin{matrix} x_1 + 2x_2 + 2x_3 = 5 \\ 2x_1 + x_3 = 4 \\ 3x_1 + x_2 + x_3 = 2 \end{matrix}$

Ans. Min $C = 40$ at $(5, 0, 0)$