

**Dawson College
Mathematics Department
FINAL EXAMINATION
201-NYC-05 section 08**

December 17, 2009 9:30- 12:30

Teachers: Jacqueline Klasa (substitute teacher Keira Ameur)

Student Name: _____

Student I.D. #: _____

Time: 3 Hours

Instructions:

- Print your name and student ID number in the space provided – above.
- All questions are to be answered directly on the examination paper in the provided space. Show your complete work and

Problem 1.- (15 marks)

Solve the following systems of linear equations using the method of your choice: either the Gauss-Jordan method or the Cramer's rule (only if applicable). For each consistent system, don't forget to tell how many solutions you found? Whenever you find infinitely many solutions give also a particular solution.

(i)
$$\begin{aligned} 3x - 7y &= 0 \\ 6x - 14y &= 12 \end{aligned}$$

(ii)
$$\begin{aligned} x - 2y + 3z &= 7 \\ 2x - 3y &= 5 \\ x - 3y + 2z &= -5 \end{aligned}$$

(iii) $x_1 - x_3 + 2x_4 = 0$
 $2x_1 + x_2 - x_3 - x_4 = 2$
 $4x_1 + x_2 - 3x_3 + 3x_4 = 2$

Problem 2.- (12 marks)

Evaluate the determinants of the following matrices and conclude about their **invertibility**:

$$\mathbf{C} = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 0 & 1 \\ 0 & 0 & 5 \end{bmatrix}$$

$$\mathbf{D} = \begin{bmatrix} 1 & 2 & 0 & 5 \\ 4 & 0 & 0 & 0 \\ 2 & -1 & 5 & 4 \\ 5 & 2 & 0 & -1 \end{bmatrix}$$

Problem 3.- (12 marks).-

Invert the following matrix A using both well known methods:

$$A = \begin{bmatrix} 1 & 3 & 5 \\ -1 & 0 & 2 \\ 4 & 4 & 2 \end{bmatrix}$$

a) Using elementary row-operations.

b) Using cofactors and the adjoint of A .

Continuation of Problem 3

c) Use the inverse found previously in problem 3 to s

Problem 4.- (10 marks)

Given the following matrices A, B and C

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \quad B = \begin{bmatrix} 7 \\ 0 \\ 1 \\ 0 \end{bmatrix} \quad C = \begin{bmatrix} 3 & 1 \\ 5 & 1 \\ 7 & 1 \end{bmatrix}$$

Calculate the following expressions (if possible):
 $5A$, A^2 , A^T , AB , AC , A^2C^2 , $(AC)^2$, $A + C$, B^TB , $\text{adj}(A)$.

Problem 5.- (15 marks)

Here A , B and C represent square matrices of same size.

a) Simplify the following expressions:

(i) $(5 B^{-1} B^{10} C^{-1} A)^{-1}$ (ii) $(5 B^{-1} (B^T)^2 A^T)^T$

b) Expand the following expression: $(A + B)(I + A^{-1})(I + B^{-1})$ where I is the identity matrix of same size as A and B .

c) If A and B are 4×4 -matrices with $\det(A) = -5$ and $\det(B) = 10$, evaluate the following determinants:

(i) $\det(BA) =$

(ii) $\det(A^{-1} B^2 A),$

(iii) $\det(2 B^{-1} A)$

(iv) $\det((10 B^2)^T)$

Problem 6.- (12 marks)

Let A, B and C be three points in the space \mathbf{R}^3 described by:

$$A = (5,1,-2) \quad B = (2,1,2) \quad \text{and} \quad C = (9,1,1) .$$

- a) Show that the triangle ABC is a right triangle. Specify where is the right angle.
- b)

Continuation of Problem 6

Problem 7.- (12 marks)

a) Let M be the point $(1, -2, 5)$ and P be the plane described by the equation :

$2x - y + 4z = 10$. Find the distance between M and the plane P .

b) Show that the line L described by the parametric equations:

$$X = 2 - t$$

$$Y = 3t$$

$$Z = 3 - 10t$$

Intersects the previous plane P at a point Q . Find the coordinates of Q .

c) Show that the new line L' described by the parametric equations:

$$X = u$$

$$Y = 1 - u$$

$$Z = 3 + 2u$$

Intersects the previous line L at a point R . Find the coordinates of R .

Problem 8.- (12 marks)

Here is the diagram of the traffic network around

Continuation of Problem 8